

# AP Stats Webinar

## Sampling Distributions of Means and Proportions

Monday, 16 Jan 2016, 7:30-8:30PM EDT

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Select **call using computer**.
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# Sampling Distributions of Means & Proportions

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# Brief Resume' (My Apologies)

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- Member of the ASA/NCTM Joint Committee on Curriculum in Statistics (2016-2019)
- AP Statistics Exam Reader and Table Leader
- ASA National Poster Competition judge (2011-Present)
- Contributor and supplements author for *The Practice of Statistics*, 5th edition (2013)
- Presenter of two-day *AP Statistics Teacher Workshop* (2015)
- Co-author of the College Board Curriculum Module on Random Sampling and Random Assignment (2012)
- Co-leader of *Experienced AP Statistics Teacher* workshop (2014)
- Co-leader of *Simulation Based Inference* workshop (2015)
- Various workshops and sessions at NCTM regional and annual conferences (2011-2015)

# Content Overview

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(This content is non-negotiable.)



# Sampling Distributions - General

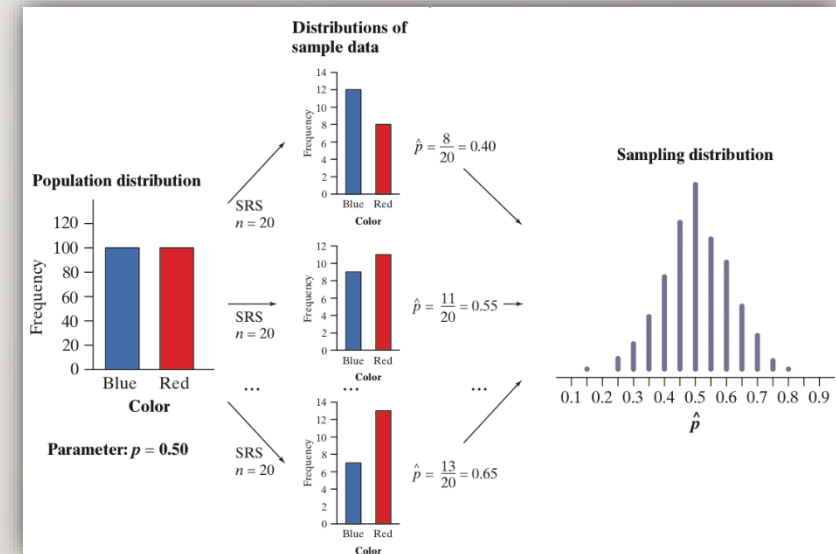
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- Sample statistics vary from sample to sample!
- Must know general ideas for all sampling distributions
  - Bias (accuracy)
  - Variability (precision)
- Must know characteristics of two specific sampling distributions
  - Sample proportions,  $\hat{p}$
  - Sample means,  $\bar{x}$



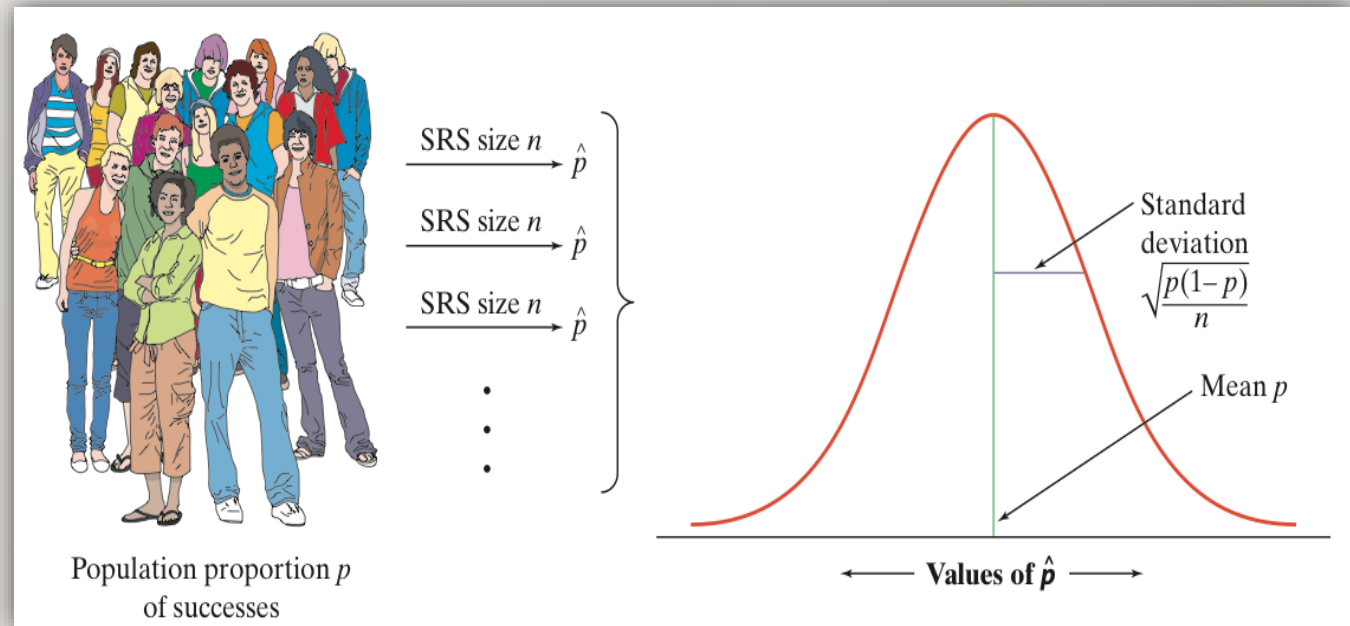
# The Sampling Distribution of a Sample Proportion $\hat{p}$

- Mean of sampling dist of  $\hat{p}$ 
  - $\mu_{\hat{p}} = p$
  - So,  $\hat{p}$  is an unbiased estimator of  $p$
- Standard deviation of sampling dist of  $\hat{p}$ 
  - $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$
  - Exactly true for sampling with replacement
  - Approximately true for sampling without replacement as long as  $n \leq \frac{1}{10}N$



# The Sampling Distribution of a Sample Proportion $\hat{p}$

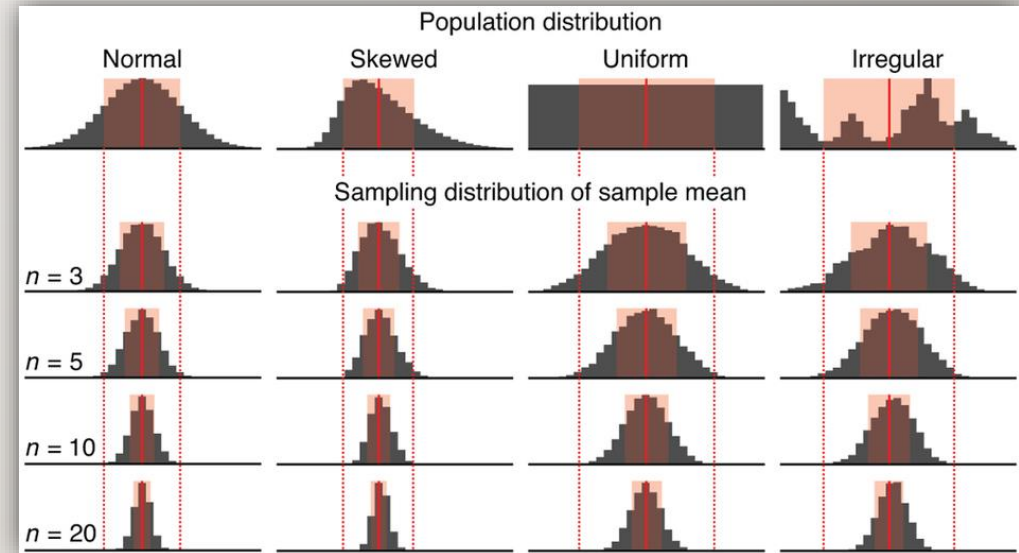
- Shape of sampling dist of  $\hat{p}$ 
  - Approximately normal if  $np \geq 10$  and  $n(1 - p) \geq 10$





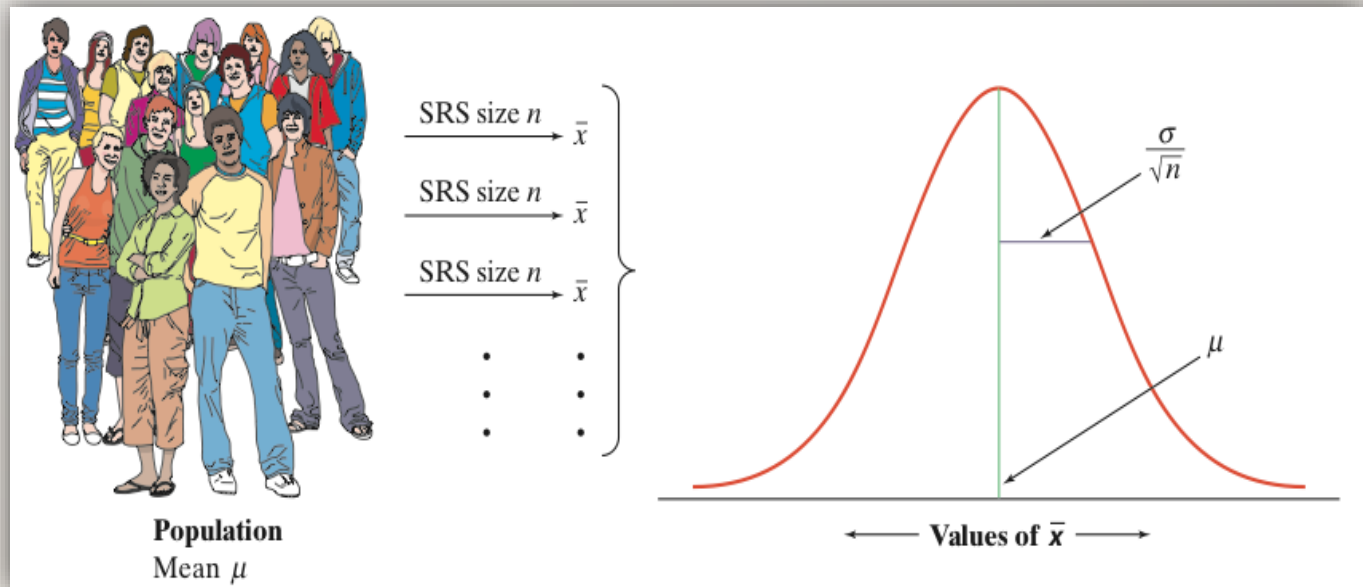
# The Sampling Distribution of a Sample Mean $\bar{x}$

- Mean of sampling dist of  $\bar{x}$ 
  - $\mu_{\bar{x}} = \mu$
  - So,  $\bar{x}$  is an unbiased estimator of  $\mu$
- Standard deviation of sampling dist of  $\bar{x}$ 
  - $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
  - Exactly true for sampling with replacement
  - Approximately true for sampling without replacement as long as  $n \leq \frac{1}{10}N$



# The Sampling Distribution of a Sample Mean $\bar{x}$

- Shape of sampling dist of  $\bar{x}$ 
  - Exactly normal for normal populations
  - Approximately normal if  $n \geq 30$  for nonnormal populations



# Teaching Ideas & Tips

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# Formula Sheet

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- The formulas for the mean and standard deviation of  $\hat{p}$  and  $\bar{x}$  are on the AP Stats Formula Sheet

If  $X$  has a binomial distribution with parameters  $n$  and  $p$ , then:

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\bar{x}$  is the mean of a random sample of size  $n$  from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

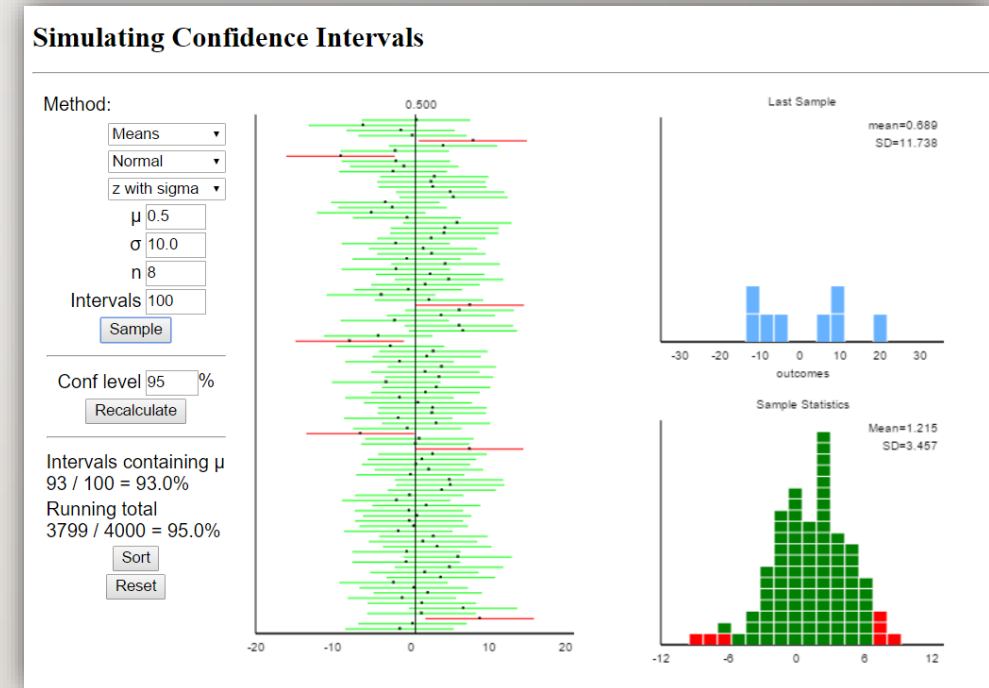
$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



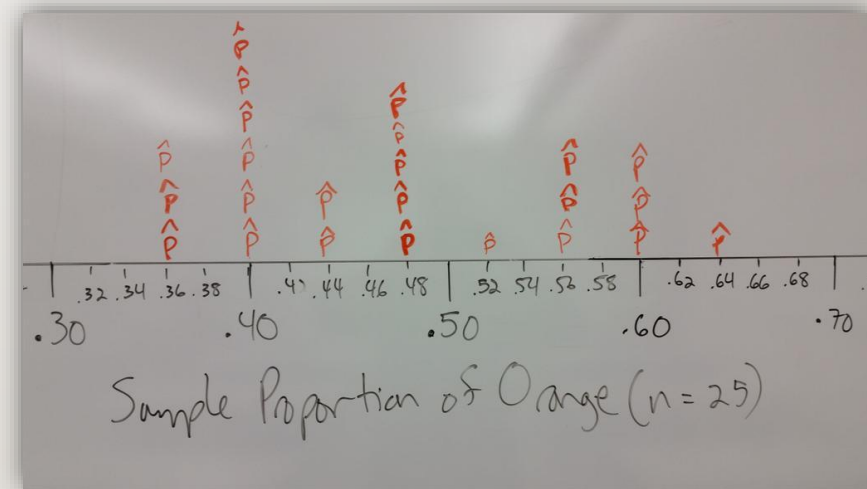
# Teaching Sampling Distributions - General

- The ideas here are the foundation for confidence intervals and significance tests
- This chapter may become more clear after CIs and STs
- Create a complete sampling distribution from a small population



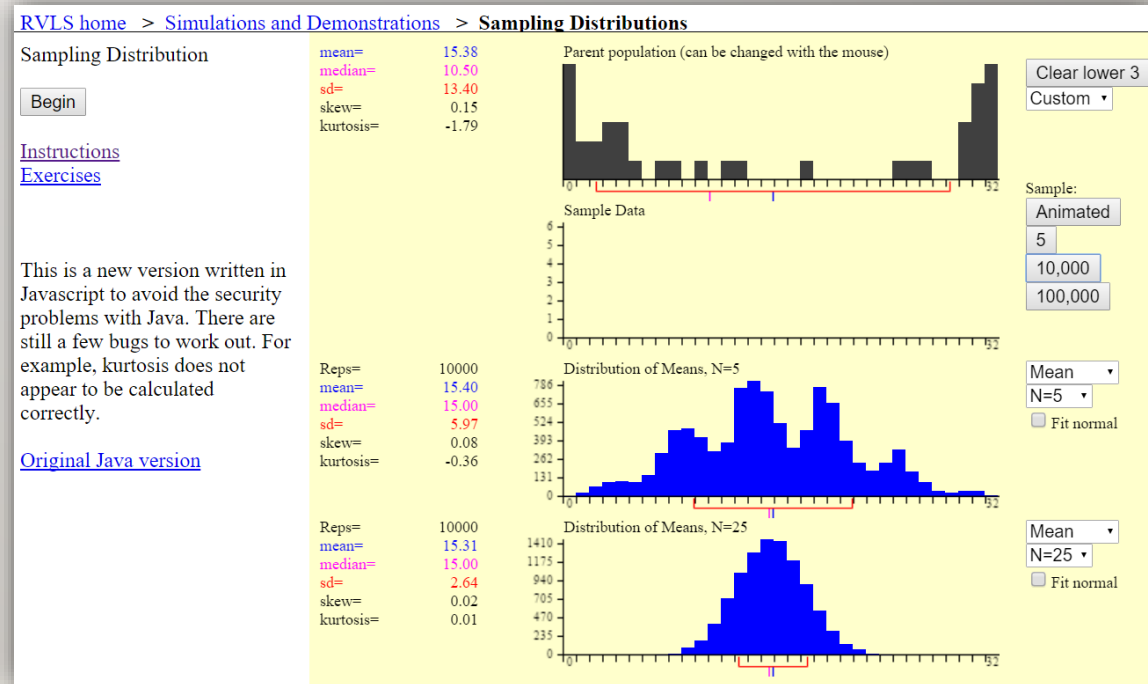
# Teaching Sampling Distributions of $\hat{p}$

- Reese's Pieces
  - Make  $\hat{p}$  plots!
  - True value of  $p$  is unknown
- Boxes with colored beads
  - Can use different values for  $p$
  - Can make (or buy) cheap paddles/scoops
- Using  $\pi$  for the population proportion is acceptable



# Teaching Sampling Distributions of $\bar{x}$

- Online Stat Book Applet
  - [http://onlinestatbook.com/stat\\_sim/sampling\\_dist/](http://onlinestatbook.com/stat_sim/sampling_dist/)
  - Spend one lesson sampling from normal populations, one from non-Normal populations
  - Look at several populations
  - Use “custom” population
  - Compare sampling distributions for different values of  $n$



# AP Exam Questions

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# 2014 APFRQ #3 – School Attendance

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3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.
- (a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?
  - (b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.
  - (c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

# 2014 APFRQ #3 – School Attendance

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- Rubric



# Multiple Choice Questions (2008)

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22. A recent survey concluded that the proportion of American teenagers who have a cell phone is 0.27. The true population proportion of American teenagers who have a cell phone is 0.29. For samples of size 1,000 that are selected at random from this population, what are the mean and standard deviation, respectively, for the sampling distribution of the sample proportion of American teenagers who have a cell phone?

(A)  $0.27, \sqrt{1000(0.27)(0.73)}$

(B)  $0.27, \sqrt{\frac{(0.29)(0.71)}{1000}}$

(C)  $0.27, \sqrt{\frac{(0.27)(0.73)}{1000}}$

☒ (D)  $0.29, \sqrt{\frac{(0.29)(0.71)}{1000}}$

(E)  $0.29, \sqrt{1000(0.29)(0.71)}$



# Multiple Choice Questions (2008)

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34. A recent study was conducted to investigate the duration of time required to complete a certain manual dexterity task. The reported mean was 10.2 seconds with a standard deviation of 16.0 seconds. Suppose the reported values are the true mean and standard deviation for the population of subjects in the study. If a random sample of 144 subjects is selected from the population, what is the approximate probability that the mean of the sample will be more than 11.0 seconds?
- (A) 0.1151
  - ☒ (B) 0.2743
  - (C) 0.7257
  - (D) 0.8849
  - (E) Based on the values of the true mean and true standard deviation, it can be concluded that the population distribution is not normal and therefore the probability cannot be calculated.



# Multiple Choice Questions (2012)

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37. There were 5,317 previously owned homes sold in a western city in the year 2000. The distribution of the sales prices of these homes was strongly right-skewed, with a mean of \$206,274 and a standard deviation of \$37,881. If all possible simple random samples of size 100 are drawn from this population and the mean is computed for each of these samples, which of the following describes the sampling distribution of the sample mean?

- ☒ (A) Approximately normal with mean \$206,274 and standard deviation \$3,788
- (B) Approximately normal with mean \$206,274 and standard deviation \$37,881
- (C) Approximately normal with mean \$206,274 and standard deviation \$520
- (D) Strongly right-skewed with mean \$206,274 and standard deviation \$3,788
- (E) Strongly right-skewed with mean \$206,274 and standard deviation \$37,881

# Multiple Choice Questions (2013)

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11. Let  $X$  be a random variable that has a skewed distribution with mean  $\mu = 10$  and standard deviation  $\sigma = 10$ . Based on random samples of size 400, the sampling distribution of  $\bar{x}$  is
- (A) highly skewed with mean 10 and standard deviation 10
  - (B) highly skewed with mean 10 and standard deviation 5
  - (C) highly skewed with mean 10 and standard deviation 0.5
  - (D) approximately normal with mean 10 and standard deviation 10
  - ☒ (E) approximately normal with mean 10 and standard deviation 0.5

# Contact Me

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